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What Is Quantum Cognition, and How Is It Applied to Psychology?

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Abstract
Quantum cognition is a new research program that uses mathematical principles from quantum theory as a framework to explain human cognition, including judgment and decision making, concepts, reasoning, memory, and perception. This research is not concerned with whether the brain is a quantum computer. Instead, it uses quantum theory as a fresh conceptual framework and a coherent set of formal tools for explaining puzzling empirical findings in psychology. In this introduction, we focus on two quantum principles as examples to show why quantum cognition is an appealing new theoretical direction for psychology: *complementarity*, which suggests that some psychological measures have to be made sequentially and that the context generated by the first measure can influence responses to the next one, producing measurement order effects, and *superposition*, which suggests that some psychological states cannot be defined with respect to definite values but, instead, that all possible values within the superposition have some potential for being expressed. We present evidence showing how these two principles work together to provide a coherent explanation for many divergent and puzzling phenomena in psychology.

Keywords
quantum cognition, complementarity, superposition, context effects, interference effects, law of total probability

Quantum mechanics is arguably the most important and best empirically confirmed scientific theory in human history. It is "essential to every natural science," and more than one third of our current global economy is based on its practical applications (Rosenblum & Kuttner, 2006, p. 81). At first glance, it may seem unthinkable to draw connections between this micro theory of particle physics and the macro phenomena observed in human cognition. Yet there are compelling scientific reasons for doing so. The emerging research program, called *quantum cognition*, is motivated by a myriad of puzzling findings and stubborn challenges accumulated over decades in psychological science. This new program has rapidly grown and exhibited the exciting potential of quantum theory to provide coherent and principled answers to the puzzles and challenges (Busemeyer & Bruza, 2012; Wang, Busemeyer, Atmanspacher, & Pothos, 2013).

Classical Versus Quantum Models of Cognition

The cognitive revolution in the 1960s was based on classical computational logic; the connectionist/neural-network movements in the 1980s and 1990s were based on classical dynamic systems; and probabilistic models of cognition that emerged in the 2000s were based on classical probability theory. These classical assumptions remain at the heart of traditional cognitive theories. They are so commonly and widely applied that we take them for granted and presume them to be obviously true. What are these critical but hidden assumptions upon which all traditional cognitive theories rely? How does quantum theory depart from these assumptions in fundamental ways? Below, we consider two examples (see Busemeyer & Bruza, 2012, for a comprehensive review).

Classical logic obeys the *commutative axiom*. Consider the proposition G, "a defendant is guilty," which can be true or false, and the proposition P, "a defendant should be punished," which can be accepted or rejected. The commutative axiom states that the order of the two propositions does not matter when you consider them; in this

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Table 1. The Quantum Principles of Complementarity and Superposition

	Complementarity	Superposition
Basic idea	Psychological measures, such as judgments, often require one to take different perspectives, which have to be taken sequentially, and the context generated by the first measure disturbs subsequent ones.	To be in a superposition state means that all possible measurement values have some potential for being expressed at each moment. The potentials can interfere with each other—like wave interference—to change the final observed measurement value. The concept provides an intrinsic representation of the conflict, ambiguity, or uncertainty that people experience.
History	It is interesting to note that Niels Bohr, who introduced the concept of *complementarity* to physics, actually imported this concept from psychology, where it was first coined by William James. It was Edgar Rubin, a Danish psychologist, who acquainted Bohr with the idea (Holton, 1970).	The ontological interpretation of the superposition state has produced volumes of philosophical debate over the past century. Perhaps one of the clearest statements is provided in Heisenberg (1958).

case, $(G \wedge P) = (P \wedge G)$.[1] As a consequence of the commutative axiom, classical probability follows the joint-probability rule: $p(G \cap P) = p(G) \cdot p(P | G) = p(P) \cdot p(G | P) = p(P \cap G)$. That is, the probability of concluding that a defendant is guilty and deciding to punish the defendant should be the same as the probability when the questions are considered in the opposite order.

In contrast, quantum theory recognizes the possibility that judgments about some attributes, such as guilt and punishment, may be *complementary*. In other words, they require examination from different perspectives or points of view, which cannot be done simultaneously (see Table 1). Instead, complementary questions need to be examined sequentially, and the answer to the first question (e.g., about guilt) produces a context that changes the answer to the next question (e.g., about punishment). Therefore, quantum theory does not necessarily obey the commutative rule, and it does not necessarily obey the classical joint-probability rule. Instead, it says that the probability of G and then P does not equal the probability of P and then G: $p(\text{G and then P}) \neq p(\text{P and then G})$. This means that when questions are complementary, their order changes the probabilities of their respective answers. In fact, quantum theory was originally developed to account for noncommutative and order-dependent measurements in physics.

Classical logic is also based on the *distributive* axiom, which, in the current context, could be formulated as follows: $P = (G \wedge P) \vee (\sim G \wedge P)$. Using our earlier example, this means that there are only two ways to conclude that a defendant should be punished (P): Either the defendant is guilty (G) or the defendant is not guilty (~G). As a consequence of this axiom, classical probability theory obeys the law of total probability: $p(P) = p(G \cap P) + p(\sim G \cap P) = p(G) \cdot p(P | G) + p(\sim G) \cdot p(P | \sim G)$. In words, the probability that a punishment is chosen should equal the probability that the defendant is thought to be guilty and a punishment is chosen, plus the probability that the defendant is thought to be not guilty and a punishment is chosen.

Quantum theory, however, does not necessarily obey the distributive axiom—a judge can decide in favor of punishment while at the same time remaining *superposed* (i.e., unresolved, indefinite, or uncertain) with respect to the question of guilt (see Table 1). Because of this superposition state, quantum probabilities allow violation of the classical law of total probability: The probability of deciding to punish when not asked to resolve the question of guilt (i.e., remaining in superposition) may differ from the total probability of deciding to punish after resolving the question of guilt (which collapses the superposition state to a definite answer, guilty or not guilty).

Evidence for Quantum Cognition: Measurement Order Effects in Psychology

Order effects are prevalent in psychological studies (e.g., Trueblood & Busemeyer, 2011; Wang & Busemeyer, 2013), but can they be predicted by the quantum principle of complementarity? Using this principle, we were able to derive a strong a priori prediction, which is called the *quantum question (QQ) equality* (Wang & Busemeyer, 2013; Wang, Solloway, Shiffrin, & Busemeyer, 2014). This precise prediction can be used to test whether quantum cognition provides a good theory for question order effects.

Suppose a person is asked a pair of questions (Questions A and B) in two different orders (AB vs. BA). Define $p(\text{Ay, Bn})$ as the probability of saying "yes" to question A and then "no" to question B, and define $p(\text{Bn, Ay})$ as the probability of saying "no" to question B and then "yes" to question A (similar notation applies to other combinations of answers). An order effect occurs when these two probabilities are not equal—$p(\text{Ay, Bn}) \neq p(\text{Bn, Ay})$—and similarly for other answer combinations. Even

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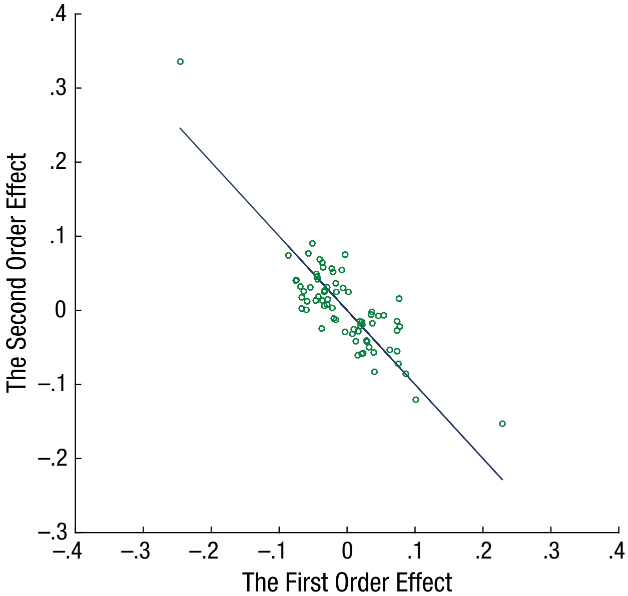

Fig. 1. Scatterplot of order effects from 70 field experiments with U.S. national representative samples (see Wang, Solloway, Shiffrin, & Busemeyer, 2014, for details). Each data point represents the results from one of the 70 experiments. The difference in the probability of Question A being answered with a "yes" and Question B being answered with a "no" when the questions were asked in an AB versus BA order is plotted on the horizontal axis; the difference in the probability of Question A being answered with a "no" and Question B being answered with a "yes" when the questions were asked in an AB versus BA order is plotted on the vertical axis. The quantum question (QQ) equality predicts that all points should fall on the line shown in the figure (i.e., intercept = 0, slope = −1). Supporting QQ equality, the correlation between these order-effect pairs was −.82.

if order effects occur, the quantum question order model predicts the following QQ equality: [p(Ay, Bn) − p(Bn, Ay)] = −[p(An, By) − p(By, An)].

This is an a priori, parameter-free, and precise prediction about the pattern of order effects, and such strong tests of theories are not common in psychology. Recently, Wang et al. (2014) showed that the QQ equality was indeed statistically supported across a wide range of 70 field experiments using national representative samples in the United States (with more than 1,000 participants in most surveys). In each of the experiments, a random half of the participants answered a pair of questions in the AB order and the other half in the BA order.

Figure 1 illustrates one way to evaluate the QQ equality. Each point in the figure represents the results obtained from one of the 70 field experiments. For each experiment, the difference in the probability of A being answered with a "yes" and B being answered with a "no" when the questions were asked in an AB versus BA order—[p(Ay, Bn) − p(Bn, Ay)]—is plotted on the horizontal axis, and the difference in the probability of A being answered with a "no" and B being answered with a "yes" when the questions were asked in an AB versus BA order—[p(An, By) − p(By, An)]—is plotted on the vertical axis. The QQ equality prediction means the data points should fall on a line with an intercept of 0 and a slope of −1. As shown, the data agree with this strong prediction closely.

Of course, quantum theory is not the only explanation for order effects. In particular, the popular anchoring and adjustment heuristic is often evoked to explain order effects. However, traditional models for this heuristic fail to account for the order effects found in the 70 national field experiments (see Supplementary Information in Wang et al., 2014). Also, when direct comparisons are made in large laboratory experiments examining order effects on inference, the quantum model predicts order effects more accurately than a traditional anchoring and adjustment model (Trueblood & Busemeyer, 2011).

Evidence for Quantum Cognition: Interference Effects in Psychology

The classical law of total probability is important for psychological theories, as it provides a foundation for Bayesian and Markov models of cognition. Violations of this law are called *interference effects*, which occur when the probability of a single event A, p(A), estimated alone in one condition differs from the total probability, $p(A \cap B) + p(A \cap \sim B)$, in another condition when A is considered along with another event, B. Interference effects have been found in numerous experiments across a wide range of topics. Here, we present one example by Busemeyer, Wang, and Lambert-Mogiliansky (2009), who compared the predictions of quantum and Markov models. The Markov model predicts that the data will satisfy the law of total probability, but the quantum model generally predicts a violation of the law (i.e., an interference effect).

As illustrated in Figure 2, on each trial, participants were shown an image of a face, which varied along two features (face width and lip thickness). The participants were asked to decide whether to "attack" or "withdraw" from the faces shown. In one condition, participants were also asked to first categorize the faces as belonging to either a "good guy" or a "bad guy" group (the categorization-then-decision condition), while in another condition, they only made the action decision directly (the decision-alone condition). The participants were provided with explicit instructions about relations among facial features, categories, and actions; they were also given extensive training, with feedback, on the task. Figure 2 summarizes the basic results of the experiment for the "bad guy" face stimuli in the study.

As shown in the figure, contrary to the Markov model, but as predicted by the quantum model, the law of total

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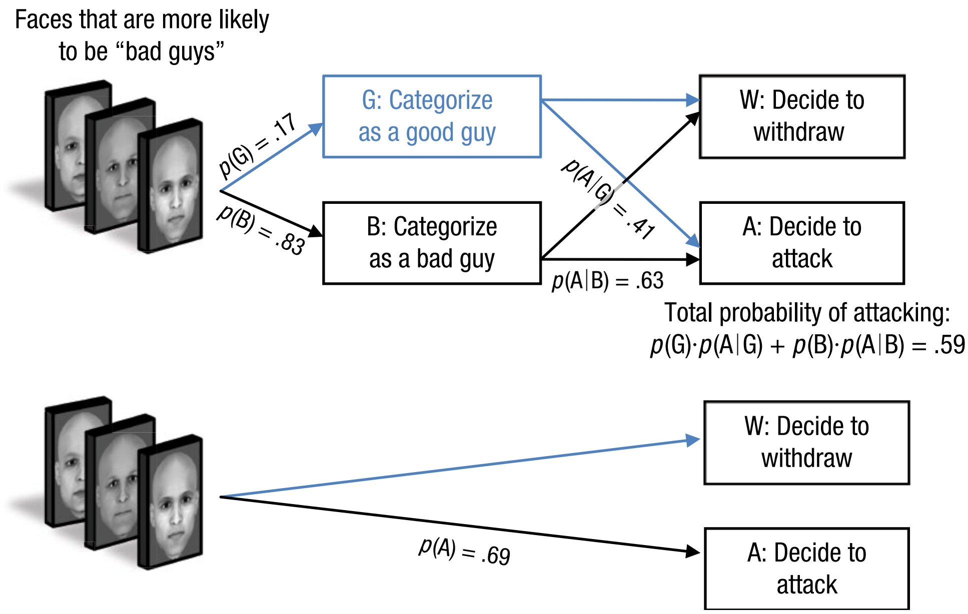

Fig. 2. Illustration of the categorization-decision experimental paradigm and interference effects (Busemeyer, Wang, & Lambert-Mogiliansky, 2009). The top path illustrates the categorization-then-decision condition, and the bottom path illustrates the decision-alone condition. The results shown are for face stimuli that participants were told were more likely to be "bad guys" (see Busemeyer et al. 2009, for details). In the categorization-then-decision condition, $p(\text{G})$ and $p(\text{B})$ are the probabilities of categorizing a face as "good" or "bad," respectively, and $p(\text{A}|\text{G})$ and $p(\text{A}|\text{B})$ are the probabilities of "attacking" (vs. "withdrawing") when the face has been categorized as good or bad, respectively; thus, the total probability of attacking is $p(\text{G})\cdot p(\text{A}|\text{G}) + p(\text{B})\cdot p(\text{A}|\text{B})$, which is .59. This is significantly lower than $p(\text{A})$, the probability of attacking, in the decision-alone condition.

probability was violated and an interference effect occurred. In the categorization-then-decision condition, the total probability of attacking was significantly lower than the probability of attacking in the decision-alone condition (.59 vs. .69). In fact, the probability of attacking after the face had been categorized as a "bad guy" was even lower than in the decision-alone condition (.63 vs. .69)!

According to our quantum model, in the decision-alone condition, participants were not required to make up their minds about a face's category, so they could remain superposed with respect to the category when deciding which action to take. This superposition state allowed interference to occur between the two thought paths regarding the category, analogous to interfering waves. Like waves, the interference can be either destructive (canceling) or constructive (resonating). In contrast, in the categorization-then-decision condition, the categorization task forced participants to resolve the indeterminacy about the category, which made the superposition collapse onto a particular category, thereby eliminating the interference between thought paths and changing the probability of attacking.

A critic might argue that quantum theory fits data better for examples such as question order effects and categorization-decision interferences simply because it is more complex. However, this has been disproved. For example, when a quantum model was compared to one of the most successful traditional decision models using a state-of-the-art Bayesian model comparison method, the Bayes factor overwhelmingly supported the quantum model (Busemeyer, Wang, & Shiffrin, 2015). This suggests that the quantum model can account for the empirical data better not because it is more complex but because it is more robust.

How Does a Quantum Cognitive Model Work?

To get a glimpse of how quantum theory works for explaining paradoxical findings in psychology, here is an example involving the conjunction fallacy. In a famous demonstration, Tversky and Kahneman (1973) presented participants with a story about a hypothetical person, Linda, that made her seem very much like a feminist. Participants were then asked to evaluate the probability of statements about Linda, including that she was a bank teller (B), that she was a feminist (F), and that she was a feminist and a bank teller ($\text{F} \wedge \text{B}$). Participants typically judged the conjunction ($\text{F} \wedge \text{B}$) to be more likely than the

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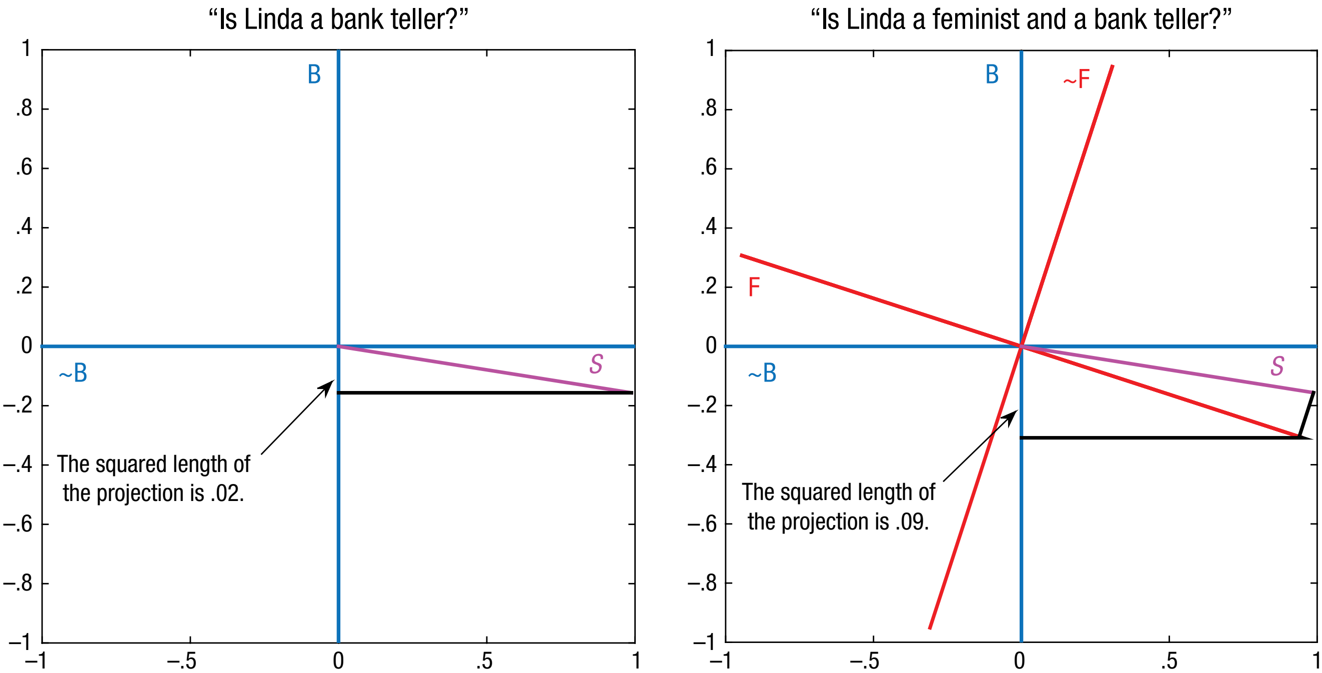

Fig. 3. A simple "toy" example of the quantum model accounting for the conjunction fallacy. The panel on the left represents the condition in which the question "Is Linda a bank teller?" is asked. The vertical blue axis labeled "B" represents the event "Linda is a bank teller," and the orthogonal blue axis labeled "~B" represents the event "She is not a bank teller." The magenta vector labeled "S" represents the person's belief based on a story about Linda. In quantum theory, the squared length of the projection of the vector S onto the axis B represents the probability of saying "yes" to the question "Is Linda a bank teller?"—which equals .02 in this example. The panel on the right represents the condition in which the question "Is Linda a feminist and a bank teller?" is asked. The negatively orientated red axis labeled "F" represents the event "Linda is a feminist," and the orthogonal red axis labeled "~F" represents the event "She is not a feminist." Once again, the magenta vector labeled "S" represents the person's beliefs based on the story. The probability of thinking "yes" to the question "Is she a feminist?" and then "yes" to question "Is she a bank teller?" is obtained by first projecting on the axis F, then projecting on the axis B, and taking the squared magnitude, which equals .09. This is larger than the probability of Linda's being a bank teller, so the result reproduces the conjunction fallacy.

event B alone, which violates the law of total probability, $p(B) = p(F \cap B) + p({\sim}F \cap B) \geq p(F \cap B)$. How does quantum theory account for this kind of finding?

Classical probability theory is built upon set theory, while quantum probability theory (i.e., the mathematical foundation of quantum theory) is built upon geometric theory. To illustrate the basic ideas of quantum models, a simple "toy" example is shown in Figure 3 for the conjunction-fallacy example. In each panel of the figure, the magenta line represents a person's belief state after hearing the Linda story. In the left panel, it lies superposed (as a linear combination) with respect to the two axes representing answers to the bank-teller question. When evaluating the question "Is she a bank teller?" the probability of saying "yes" is the squared projection from the belief state to the bank-teller axis, which equals .02 in this example. In the right panel, the same belief state is also superposed (as a linear combination) with respect to a new perspective—as shown by the rotated axes—for the feminist question. When evaluating the question "Is she a feminist and a bank teller?" the probability of saying "yes" equals the square of the product of projections from feminist and then to bank teller, which is .09 in this example. The latter is greater than the former, reproducing the conjunction fallacy. It is interesting to note that this same geometry reproduces the disjunction fallacy (Busemeyer, Pothos, Franco, & Trueblood, 2011), in which case the statement "She is a feminist" is judged to be more likely than the statement "She is a bank teller or a feminist." In fact, in addition to conjunction and disjunction fallacies, the same quantum-cognition model illustrated using the "toy" example here explains a range of other psychological phenomena, including the order effects and interference effects discussed earlier, as well as asymmetric similarity judgments (Pothos, Busemeyer, & Trueblood, 2013) and "irrational" decision making (Pothos & Busemeyer, 2009).

Concluding Comments

Although quantum cognition is a new field, interest in it is growing rapidly. Recent new applications attack a diverse range of challenging problems in psychology, including bistable perception (Atmanspacher & Filk, 2010), overdistribution in episodic memory (Brainerd, Wang, & Reyna, 2013), entanglement in associative memory (Bruza, Kitto,

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Nelson, & McEvoy, 2009), violations of rational decision making (Pothos & Busemeyer, 2009; Yukalov & Sornette, 2011), probability judgment errors (Busemeyer et al., 2011), over- and under-extensions in conceptual combinations (Aerts, Gabora, & Sozzo, 2013), order effects on inference (Trueblood & Busemeyer, 2011) and causal reasoning (Trueblood & Busemeyer, 2012), asymmetric similarity judgments (Pothos et al., 2013), and vagueness (Blutner, Pothos, & Bruza, 2013). The power of quantum cognition comes from the research program's adherence to a small core set of coherent principles for deriving formal models that tie together various and myriad findings from widely different topics in psychology.

Recommended Reading

Busemeyer, J. R., & Bruza, P. D. (2012). (See References). A book that serves as an introduction to and tutorial on quantum cognition by a psychologist and computer scientist and that reviews several applications of the theory to different areas of psychology in great depth.

Khrennikov, A. Y. (2010). *Ubiquitous quantum structure: From psychology to finance*. Berlin, Germany: Springer. A book by a mathematical physicist that provides an introduction to applications of quantum theory outside of physics and also describes applications to decision making.

Haven, E., & Khrennikov, A. (2013). *Quantum social science*. Cambridge, England: Cambridge University Press. A book by an economist and a physicist that provides an introduction to quantum theory with applications to a broader range of social sciences, including economics and finance.

Pothos, E. M., & Busemeyer, J. R. (2013). Can quantum probability provide a new direction for cognitive modeling? *Behavioral & Brain Sciences*, *36*, 255–274. An article that provides a short introduction to and review of the research on quantum cognition and compares it to other probabilistic models of cognition.

Wang, Z., Busemeyer, J. R., Atmanspacher, H., & Pothos, E. M. (2013). (See References). An article that presents definitions of key concepts in quantum cognition and provides an introduction to a special issue of *Topics in Cognitive Science* containing several articles presenting applications of quantum theory to cognition (e.g., memory, attitude, concepts, and decisions).

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Note

1. Notations used in the article are summarized here. Symbols for logic: Define A and B as two events; then $(A \wedge B)$ means "A and B," $(A \vee B)$ means "A or B," and ~A means "not A." Symbols for probability: Define $p(A)$ as the probability that event A occurs and $p(B)$ as the probability that event B occurs; then $p(A \cap B)$ is the probability that both event A and event B occur, $p(A \cup B)$ is the probability that at least one of the two events occurs (i.e., A occurs, B occurs, or both A and B occur), and $p(\sim A)$ is the probability that A does not occur; in addition, conditional probability $p(B|A)$ is the probability that B occurs given that A has already occurred.

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